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THE RADIUS OF CURVATURES OF THE CORNEA.

—BY—

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THE RADIUS OF CURVATURES OF THE CORNEA.

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The classification of the sciences has at various times engaged the attention of philosophers. Whether or not the thoughts of students of biology have ever been directed to the merits of the classifications, among others, of Bacon,[1] Comte,[2] Spencer,[3] the most superficial thinker must have been impressed by the fact that whatever be the logical order of the study of the sciences, the progress of any one branch of science has depended largely upon the simultaneous advance made in some closely or even remotely allied branch.

The influence exerted by the progress of physics upon that of astronomy and biology through the invention of the telescope and microscope respectively, the stimulus to mathematical research excited by astronomical discoveries, the influence of physics upon chemistry, may be cited as familiar examples. As a special instance of the manner in which the different branches of science are related to each other, it may be mentioned that the method of measuring the radius of curvature of the cornea by means of the ophthalmometer is essentially the same as that made use of by astronomers in determining the angle subtended by the apparent diameter of a celestial body by means of the heliometer.

Though not perhaps generally known, there was submitted as long ago as 1743 to the Royal Society a communication by Savery, giving an account of his invention of "A new way of measuring the difference between the apparent diameter of the sun .... with a micrometer placed in a telescope invented for that purpose." Savery's paper was not, however, published at that time, it having been apparently entirely forgotten by Bradley to whom it was referred. Ten years afterward, during the year 1753, Short having learned that Bouguer had given an account in 1748 to the Academy of Sciences of Paris[4] of his invention of the heliometer, and wishing to secure for his countryman the credit of the invention, called the attention of the Royal Society, in whose Transactions[5] it finally appeared, to the long neglected communication of Savery.

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[1] De Augmentis Scientiarum.
[2] Cours de Philosophie Positive.
[3] Genesis of Science Essays, 1868.
[4] Historie de L'Academie Royale des Sciences a Paris, MDCCLII, pp. 87, Memoires p. 11.
[5] Philosophical Transactions, Vol. XLVIII, p. 165.

The construction of the heliometer is based upon the fact that while an object appears single (Fig. 1) when viewed through two object glasses of equal focal length placed side by side at the end of a tube provided with a single eye piece, it will appear double (Fig. 2) when the object glasses are displaced. As the two images (Fig. 2) at the focus are separated from each other by a distance equal to the distance of the centres of the object glasses, the displacement of the latter as measured by a micrometer screw will give the displacement of the angular diameter, (Fig. 2, c d) of a celestial body. The angle subtended by the apparent diameter of the sun, for example Fig. 2, c d, can then be determined by simply bringing the two images of the sun in contact (Fig. 2) and measuring the displacement necessary to accomplish this by a filar micrometer, the angular value of the revolution of the screw being known.

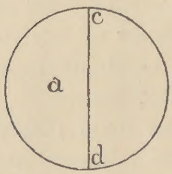

Fig. 1.

A few years after Bouguer invented his heliometer which Lalande used to determine the diameter of the sun, Dollond substituted for the two object glasses the two halves of one object glass (Fig. 3, A B). By means of this arrangement two images (Fig. 2, a b) are seen as in Bouguer's heliometer, each semi-lens giving an image except when the two semi-lenses are juxtaposed, the two images being then superimposed. By fixing the semi-lens A and making the semi-lens B(Fig. 3)slide over it, the image a (Fig. 2) due to the semi-lens A becomes also immovable. The distance through which the semi-lens B (Fig. 3) is moved in order to bring the image b (Fig. 2) due to it, in contact tangentially with the image a and equal to the apparent diameter sought, is measured by means of a micrometer screw as in Bouguer's instrument. The value of the angle under which the apparent diameter of the object (Fig 2, c d) is seen can be thus determined.

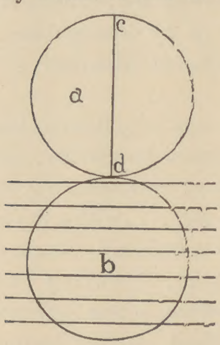

Fig. 2.

In connection with the invention of the heliometer, it is interesting as illustrating the influence of the progress of astronomy upon physiology that the Observatory of Königsberg, to which Helmholtz was attached as assistant when a young man, possesses one of the finest heliometers ever constructed: that by Fraunhofer. Doubtless this instrument suggested to Helmholtz the invention of the ophthal-

mometer. It should be mentioned, however, that in the ophthalmometer as devised by Helmholtz[6] the two images of the object are produced by means of two glass plates such as were suggested by Clausen to obtain two images of a star. The ophthalmometer as constructed by Meyerstein of Göttingen consists of a small telescope (Fig. 4, T) the objective of which, b, is enclosed within a brass box C, the latter supported upon a tripod and capable of being leveled in all azimuths. The brass box contains two plane parallel glass plates P^1 P^2, which are situated in front of the objective of the telescope. By means of a screw mechanism the glass plates can be so turned around a common axis that they can be brought either into the same plane (Fig 5) or into two different planes (Fig. 6). The angular deviation produced by the movement of the glass plates just referred to can be determined by two graduated wheels which rotate as the glass plates diverge from each other to the one-tenth of a degree, the instrument being provided with a vernier.

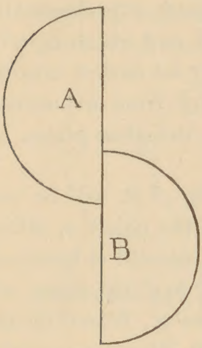

Fig. 3.

By means of the ophthalmometer we are enabled to determine the size of the virtual image produced when rays of light emanating

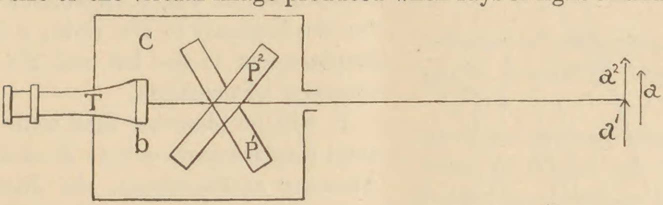

Fig. 4.

from an external object are allowed to fall upon the cornea and, further, as we will show presently, by determining from the size of such corneal image the radius of the curvature of the cornea. The principle by which the size of the corneal image is determined by the ophthalmometer is the same as already mentioned as that of obtaining the apparent diameter of a heavenly body by means of a heliometer. The only essential difference in the theory of the two instruments is that while in the heliometer one semi-lens alone moves, and moves in the same plane with the other fixed semi-lens, (the displace-

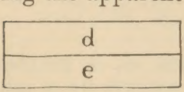

Fig. 5.

[6] Archiv fur Ophthalmologie. Band 1, Abth. II, S 1.

ment being measured by a micrometer screw), in the ophthalmometer both glass plates move, and move in different planes, the displacement being deduced from the formula $I=2\,T\,\frac{\sin(a-b)}{\cos b}$ in which a=the angular deviation of the glass plates obtained experimentally. Before developing the formula just mentioned and which is indispensable for the working of the ophthalmometer let us first consider the paths of such rays of light as, emanating from a luminous object or its reflected image, pass through its two glass plates, the latter making an angle with each other.[7]

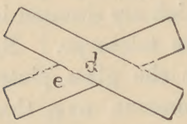

Fig. 6.

From a consideration of Fig. 7 it will be seen that as the rays of light from the object a, falling upon the glass plate P^1 are projected subjectively back to a^1 those falling upon P^2 to a^2 the object will be consequently seen double as a^1 a^2. Now if the two images a^1 a^2 are brought into contact with each other tangentially, their total displacement, that is the sum of the displacement of the two images to the right and to the left, will be equal to the size of the object. That such must be the case becomes at once apparent, if the object and the two images are round bodies as represented in Fig. 8, in which a b is the displacement to the right, c d the displacement to the left and a d the diameter of the object.

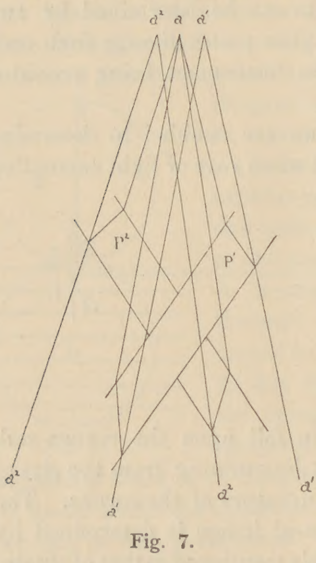
Fig. 7.

It will be observed that while the total displacement—a b+c d=a d the diameter of the object, the distance c b between the outer edges of the two images is twice a d, or twice the diameter of the object.

Such being the case it is evident that if we can measure the total displacement of the two images of the object when the images are in contact we will then obtain the size of the object.

[7] It is unnecessary to say that as long as the two glass plates are in the same plane the luminous object being seen as a single image it is unnecessary to consider the paths of the rays of light under these conditions.

Let us consider now the development of the formula, $I = 2T \frac{\sin (a-b)}{\cos b}$ in which I is the total displacement, T the thickness of the glass plates, a the angle of incidence or angular deviation, b the angle of refraction. Let A B C D (Fig. 9,) be one of the glass plates placed in front of the objective of the telescope and a c an incident ray emanating from the image of the luminous object whose size is to be determined. Such being the case the ray a c, according to the law of refraction, will be bent as c I toward the perpendicular u K as it passes through the glass and away from the perpendicular as I l and parallel with a c as it emerges from the glass. The angle a c u will be equal to the angle l I m, parallel rays falling upon parallel surfaces. Calling the angle of incidence a c u, A, and the angle of refraction I c K, B, we will have $\frac{\sin A}{\sin B}$ = index of refraction = 1·524 or $\sin A = \frac{\sin B}{1{\cdot}524}$, from which equation we can obtain the value of B and cos B from trigonometric tables, A being obtained by observation.

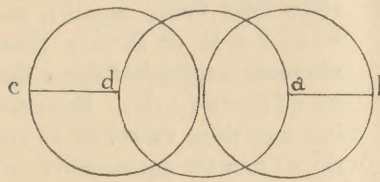

Fig. 8.

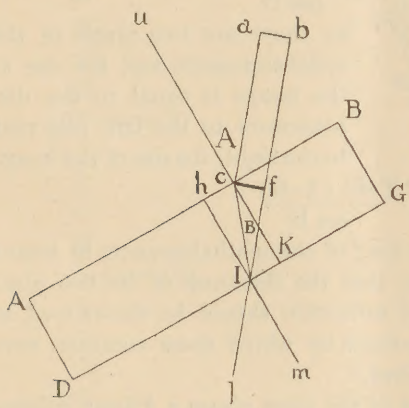

Fig. 9.

Such being the relation of the angles of incidence and refraction the point a to the eye of an observer at l would appear to be at b, the glass plate effecting therefore a lateral displacement of the point a to an extent a b which is measured by determining the equal distance c f.

Inasmuch, however, as c f = c I sin c I f, i t follows that if we can determine c I and sin c I f we will obtain c f or its equal a b. As K c = cI cos B, it follows that $cI = \frac{K\,C}{\cos B}$ and as K C = the thick-

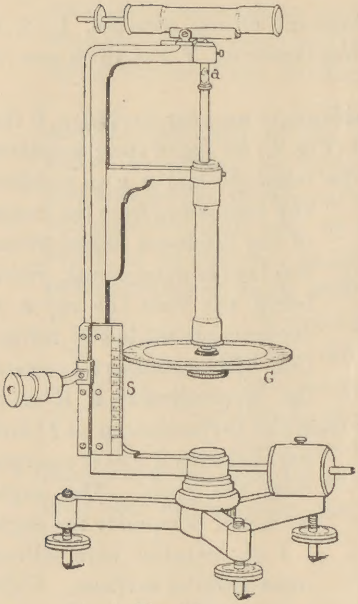

Fig. 10.

ness of the glass plates which is known, we may call it T and say therefore that CI= $\frac{T}{\cos B}$.

Further the angle c I f = h I f —h i c, but as h I f = u c a or A, their sides being parallel, and h i c=I c K or B they being alternate angles it follows that sin c I f=sin (A–B). Substituting now these values of c I and sin c I f in the equation c f=c I sin c I f we obtain c f or a b=T $\frac{\sin (A-B)}{\cos B}$. Inasmuch, however, as there are two plates in the ophthalmometer and the size of the image is equal to the displacement to the left, plus that to the right, the size of the image or I will be equal to 2 a b=2 T $\frac{\sin (A-B)}{\cos B}$

As it is essential in the working of the ophthalmometer by means of the formula just developed, that the thickness of its two glass plates as well at their index of refraction should be determined as accurately as possible, the methods by which these contrasts were obtained will be briefly described.

In determining the thickness of the glass plates a Pfister sphärometer was made use of, the general construction of which is well shown in Fig. 10. The object to be measured in this instance, the glass plate, was placed upon a little table at the top of the screw (not represented in fig.) and the latter elevated until the glass plate came in contact with the knife edge a. The level being then in adjustment, the extent of the separation of the little table and the upper knife edge or the thickness of the glass plates was then obtained by observing the distance through which the screw had descended, as given by the divisions on the scales and on the graduated circle, each division of the scale corresponding to $\frac{1}{2}$ millimeter each division of the circle to $\frac{1}{100}$ millimeter. The following resumé of 25 determinations of the thickness of each glass plate proves the accuracy of the method made use of:—

As the average thickness of the two glass plates is the same, viz.: 4·4015 mm., the result is what might have been anticipated since the two glass plates were the halves of one plate.

Resumê of 25 determinations of the thickness of the right and left glass plates of the Ophthalmometer by the Sphärometer.

Readings from the Instrument. R.	L.	Corrected and reduced. R. mm.	L. mm.
8—81·1 error 1·0	8—81·2	4·4005	4·4010
·5	·5	25	25
·4	·6	20	30
·2	·0	10	00
·0	·1	00	05
·1 error 1·0	·0	05	00
·0	·8	00	40
·8	·3	40	15
·8	·2	40	10
·0	·0	00	00
80·8 error 0·7	·0	05	15
81·1	·6	20	45
·2	·0	25	15
·3	·1	30	20
·6	·0	45	15
·1 error 0·9	·0	10	05
·1	·6	10	35
·4	·3	25	20
·2	·2	15	15
·3	·2	25	15
80·8 error 0·8	·0	00	10
·9	·1	05	15
·8	80·8	00	00
·8	·9	00	05
81·0 error 0·8	·9	10	05
		4·4015 mm.	4·4015 mm.

In determining the index of refraction of the two plates of the ophthalmometer the sphärometer was also made use of, its application for this purpose being based upon the principle of the index of refraction being equal to the ratio of the actual thickness to the apparent thickness when viewed normally to the surface. Let us suppose that a number of equidistant, parallel lines ruled upon a

glass, e. g. a stage micrometer, be viewed under the microscope and when seen in focus, that a spirit level be placed upon the upper ends of the microscope and of the screw of the sphärometer, the spirit level of the latter having been temporarily removed. The upper ends of the two instruments being now at the same level, let a reading be taken off the sphärometer and, for example, suppose it to be 46·16. Now place upon the stage micrometer one of the glass plates of the ophthalmometer, the latter being interposed between the lines and the objective of the microscope. The tube of the microscope must now be elevated in order that the lines may be brought again into focus. The upper end of the microscope being now at a higher level than that of the sphärometer, the screw of the latter must now be raised until the upper ends of the two instruments are again at the same level and a second reading be taken which we will suppose, for example, to be 43·14. The difference between the two readings, 3·02=1·510 mm.,[8] will be the distance through which the microscope was elevated in order to see the ruled lines when the plates of the ophthalmometer were interposed between the lines and the microscope. Now if this difference, 1·510 mm., be deducted from the thickness of the glass, 4·4015, and the remainder, 2·8915 mm., be divided into the thickness of the glass, the quotient, 1·5222, according to the principle above enumerated, will be the index of refraction sought. Thus, for example, let Fig. 11, represent the micrometer resting upon the stage of the microscope, A G, the actual thickness of the plate of the ophthalmometer interposed between the lines and the microscope, then according to theory and experiment A G—A B=B G or 4·4015 mm.—1·510

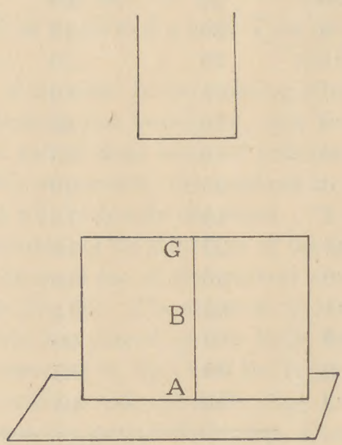

Fig. 11.

[8] Each division of the scale equals ½ of a millimeter.

NOTE.—The methods just described of determining the thickness of the glass plates of the ophthalmometer and their index of refraction were suggested to the authors by Professor A. W. Goodspeed of the University of Pennsylvania, whose well known experimental skill is a sufficient guarantee of the accuracy of the data submitted and the results obtained.

mm.$=2{\cdot}8915$, $\frac{A\,G}{B\,G}=$ index of refraction or $\frac{4{\cdot}4015 \text{ mm.}}{2{\cdot}8915 \text{ mm.}}=$ 1·5222=index of refraction.

The average index of refraction as obtained from ten experiments with each plate of the ophthalmometer by the method just described, was 1·5249, as shown by the following resumé :—

Resumé of experiments with sphärometer and microscope to determine the value of the index of refraction of the glass plates of the ophthalmometer.

R.	L.
1·5222	1·5276
1·5222	1·5276
1·5249	1·5249
1·5276	1·5249
1·5276	1·5222
1·5222	1·5222
1·5249	1·5276
1·5276	1·5249
1·5249	1·5222
1·5276	1·5222
1·5252	1·5246

1·5252
1·5246

2)3·0498

1·5249=index of refraction.

For the sake of simplicity, we have hitherto assumed that the luminous object was single, two images being then seen by means of the ophthalmometer when the glass plates of the latter were made to diverge. In the practical working of the instrument it will be found, however, that if three luminous objects be used, three gas jets for example, placed at right angles to the long axis of the telescope, six images being then perceived, the plates diverging, that it will be easier to determine the displacement of the images than if one be used. The principle involved, however, is the same whether one or three luminous objects be used. This will be made apparent by Fig. 12, in which the distance between the first image and a point midway between the second and third images corresponds to a single image, and 1 C and C 3′ to the displacement of the images to the right

and left respectively. The subject of the experiment upon whose eye the light from the three gas jets falls, sits with the head resting in a frame at a distance of about 90 centimeters from the ophthalmometer, and looks directly at the center of the frame supporting the three gas jets. Three corneal images being then perceived by the observer looking through the ophthalmometer erect virtual, but reversed as in a looking glass, the plates of the ophthalmometer are made to diverge until the image 1′ reaches a point just midway between the images 2 and 3. The original images, or the distance between 1 and a point midway between 2 and 3, being then doubled, the angle of deviation of the plates, or A, is then read off. The angle A, so obtained experimentally, being substituted in the formula $I=2\ T \frac{\sin (A-B)}{\cos B}$ the size of the image or I is then obtained by calculation as shown by the following example:

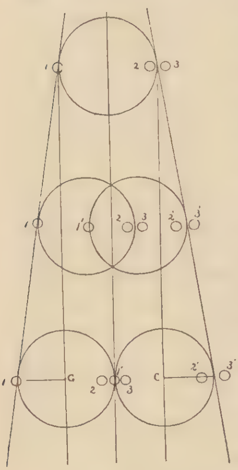

Fig. 12.

$$I=2\ T \frac{\sin (A-B)}{\cos B}$$

A=32° 48′, T=4·4015 mm. Index of refraction=1·5249

$$\frac{\sin A}{\sin B}=\text{index of refraction}=1{\cdot}5249$$

$$\frac{\sin 32° 48'}{1{\cdot}5249}=\sin B$$

log sin 32° 48′	=9·733765
log 1·5249	=0·183241
log sin B	=9·550524
B	=20° 48′ 30″

A—B=11° 59′ 30″

log sin (A—B)	=9·317582
log cos B	=9·970707
	9·346875
log 2 T	=0·944631
log I	=0·291586
I	=1·957 mm.

It has already been incidentally mentioned that the size of the corneal image having been determined, we are enabled thereby to deduce the radius of the curvature of the cornea. This is accomplished by means of the formula $d—I : I :: D : \frac{R}{2}$ or $R = \frac{2\,D\,I}{d—I}$ in which Fig. 13 R is

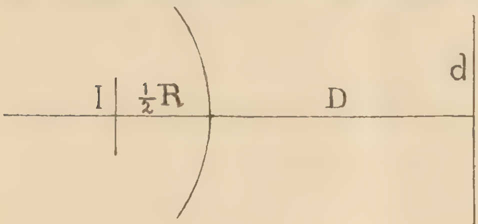

Fig. 13.

the radius sought, D the variable distance of the luminous object, the three gas jets, from the cornea C, I the size of the corneal images as determined by the ophthalmometer, and d the constant length of the luminous object 480 mm., that is the distance between the first gas jet and a point midway between the second and third gas jets, the latter being separated by a distance of 80 millimeters. The following example will illustrate the manner in which the formula is used:—

$$R = \frac{2\,D\,I}{d—I}$$

D=900 mm. d=480 mm. I=1·957 mm.

$$\begin{array}{rl} d = & 480·000 \\ I = & 1·957 \\ \hline d—I = & 478·043 \\ 2\,D = & 1800· \\ \text{Log } 2\,D = & 3·255273 \\ \text{Log } I = & 0·291506 \\ \hline & 3·546779 \\ \text{Log } (d—I) = & 2·679467 \\ \hline \text{Log } R = & 0·867312 \end{array}$$

R = 7·367 mm.=radius of curvature of the cornea.

In connection with the formulæ just made use of in determining the radius of the curvature of the cornea, it may not be superfluous to call attention to the extent to which the result so obtained will be affected by small errors made in determining the values of T, n, I, D, d, incidental to experimentation. Thus, for example, if an error of 0·02 mm. be made in T, the error in n will amount to 0·0002 mm. It is not necessary, therefore, to use the value of the thickness of the

glass plate T beyond the second decimal place, since the value of n, or the index of refraction, cannot be measured beyond the third decimal place. While an error of 2 mm. in the measurment of D will affect R to an extent only of 0·016 mm., the same amount of error in d will affect R as much as the 0·032 mm. An error of 0·02 mm. in T will involve only an error of 0·01 mm. in I, but an error of 0·01 mm. in n will change I by as much as 0·02mm. Finally as an error of 0·02 mm. in I will change the value of R by as much as 0·07 mm. it follows that of these different values that of n, or the index of refraction, is the most important, the value of n affecting that of R through I.

Resumé of 50 experiments with the ophthalmometer.

	Name.	Age.	Horizontal Meridian Radius of Curvature. mm.	Vertical Meridian Radius of Curvature. mm.
1.	A. E. H.	25	7·367	7·312
2.	J. B. L.	20	7·833	7·747
3.	E. E. S.	23	7·744	7·345
4.	W. W. K.	36	8·240	8·001
5.	C. E. P.	26	7·938	7·851
6.	W. H. S.	27	7·935	7·819
7.	C. R. C.	32	7·654	7·172
8.	W. H. L.	24	7·921	7·606
9.	A. A. E.	22	7·882	7·790
10.	G. J. M.	23	7·845	7·816
11.	E. J. C.	36	7·753	6·637
12.	S. R. H.	24	7·670	7·527
13.	S. J.	26	7·815	7·527
14.	M. W. M.	22	7·321	7.209
15.	H. O. S.	20	8·259	7·305
16.	G. S.	20	7·438	7·212
17.	G. M. K.	25	7·890	7·629
18.	M. L.	26	8·993	7·789
19.	H. L.	27	7·572	7·430
20.	O. A.	19	7·861	7.572
21.	J. N.	27	7·648	7·676
22.	C. R.	24	7·803	7·515
23.	L. L.	24	7·249	7·277
24.	M. C. S.	22	7·745	7·377
25.	F. K. P.	21	7·816	7·498
26.	G. W. H.	26	8·170	7·935

27.	H. B. K.	20	8·169	7·962
28.	J. W. K.	25	8·087	7·678
29.	B. E. H.	21	7·736	7·448
30.	W. C.	21	7·531	7·436
31.	B. L.	22	7·613	7·441
32.	A. McG.	25	8·482	8·190
33.	C. R. G.	22	7·666	7·180
34.	J. B.	25	8·286	8·166
35.	C. K. F.	21	7·449	7·336
36.	G. H. J.	19	8·122	7·799
37.	W. N. H.	23	7·233	7·012
38.	W. B. R.	44	7·841	7·637
39.	H. E. C.	25	8·013	7·399
40.	J. H. W.	27	7·670	7·384
41.	E. L. H.	22	7·799	7·596
42.	A. C.	23	7·748	7·519
43.	F. O.	20	7·760	7·361
44.	C. C.	24	7·791	7·421
45.	W. C.	23	7·659	7·433
46.	J. C.	25	7·802	7·743
47.	D. E. A.	24	7·605	7·405
48.	M. P.	21	7·803	7·345
49.	M. P. W.	27	7·744	7·572
50.	J. G.	19	7·861	7·543
	Mean	$\frac{1213}{50}=24·3$	$\frac{389·832}{50}=7·797$	$\frac{377·580}{50}=7·552$

From the above resumé it will be observed that, on the average in young men at least, the radius of curvature of the cornea amounts in the horizontal meridian to 7·797 mm., in the vertical meridian to 7·552 mm.

In conclusion it should be mentioned that the importance of determing the radius of curvature of the cornea, depends upon the fact that its value, together with that of the indices of refraction of the various refractive media of the eye, constitute the experimental data for determining the cardinal points of the eye. By means of the latter, the paths of the rays of light passing through the media of the eye, the position and size of the retinal images, etc., can be constructed or calculated, data indispensable to the comprehension of vision and the practice of ophthalmology.